PHASE NOISE AND FREQUENCY STABILITY OF VERY-HIGH FREQUENCY SILICON NANOWIRE NANOMECHANICAL RESONATORS

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Abstract: We report measurements and analyses of noise characteristics of very-high frequency (VHF) silicon nanowire (SiNW) nanoelectromechanical systems (NEMS). VHF SiNW resonators vibrating at ~200MHz typically have displacement sensitivity of ~5fm/Hz$^{1/2}$ and force sensitivity of 50~250aN/Hz$^{1/2}$, set by thermomechanical fluctuations. They have ~1nm critical amplitude and intrinsic dynamic range of 90~110dB. Amplifier noise and resistor thermal noise dominate the resonance detection, resulting in compromised displacement noise floor (typically ≥30fm/Hz$^{1/2}$), dynamic range (reduced to 70~90dB), and phase noise (≥20~30dB degradation). We develop SiNW-NEMS-based phase-locking techniques to investigate the phase noise and frequency stability performance. Frequency stability of ~0.1ppm and resonant mass sensitivity of ~10zg (1zg=10^{-21}g) have been achieved.

Keywords: Nanoelectromechanical System, Nanowire, Resonator, Phase Noise, Frequency Stability

TRANSDUCERS & EUROSENSORS '07
The 14th International Conference on Solid-State Sensors, Actuators and Microsystems, Lyon, France, June 10-14, 2007

1. INTRODUCTION

Silicon nanowires (SiNWs) made by bottom-up chemical synthesis are emerging as interesting building blocks for novel nanoscale devices. SiNW transistors for nanoelectronics [1] and SiNW sensors for biotechnology [2] have received considerable attention. With Si being a classical MEMS material, SiNWs are expected to possess excellent mechanical properties for NEMS applications. However, progress has been hindered by the difficulty of making free-standing SiNWs, until the recent development of growth processes for suspended SiNW nanomechanical devices [3]. We have recently demonstrated VHF NEMS resonators based upon such SiNWs [4] and they have exhibited very attractive properties.

As VHF resonating NEMS offer immense potential for applications ranging from resonant sensing [5] to nanomechanical signal/information processing and communication [6], assessing their noise performance is crucial. Here we present the first measurements and analyses of noise specifications of the VHF SiNW devices, and the frequency stability and phase noise performance at the system level.

2. VHF SiNW RESONATORS

Doubly-clamped SiNW resonators suspended in microtrenches, shown in Figure 1 (a), are prepared by employing a self-aligned epitaxial growth process as detailed in [3]. Figure 1 (b) displays a SiNW with conductive supporting pads ready for electromechanical characterization.

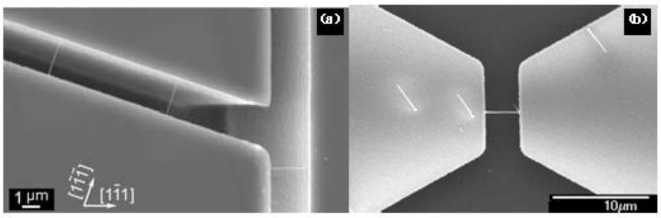

Figure 1. Scanning electron micrographs of suspended SiNWs grown across microtrenches. ***(a)*** *Oblique-view of suspended SiNWs bridging deep microtrenches.* ***(b)*** *Top-view of a typical doubly-clamped SiNW resonator with engineered supporting pads.*

VHF resonators have been demonstrated using both metalized SiNWs better matched to 50Ω (with ~30nm/5nm Al/Ti metallization), and bare pristine SiNWs with 1~100kΩ resistances (doping dependent), as described in [4]. Here we highlight the robust VHF operations of SiNW resonators in both linear and nonlinear regimes, as shown in Figure 2 (a) with the measured response from a 188MHz metalized SiNW (SiNW-M188) and in Figure 2 (b) the data from an 80MHz pristine SiNW (SiNW-80). Both kinds of SiNWs can operate into deep nonlinear regime and have

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stiffening frequency pulling. Having obtained the root-mean-square (rms) voltage signal $V(\omega)$ seen by the preamplifier in calibrated measurements, we determine the vibrating SiNW's midpoint rms displacement via $a(\omega)=V(\omega)/[\eta BL\omega_0]$ based on magnetomotive transduction [4], where η=0.5232 is the mode integral constant, B the magnetic field in Tesla, L the SiNW length, and $\omega_0=2\pi f_0$ the resonance frequency.

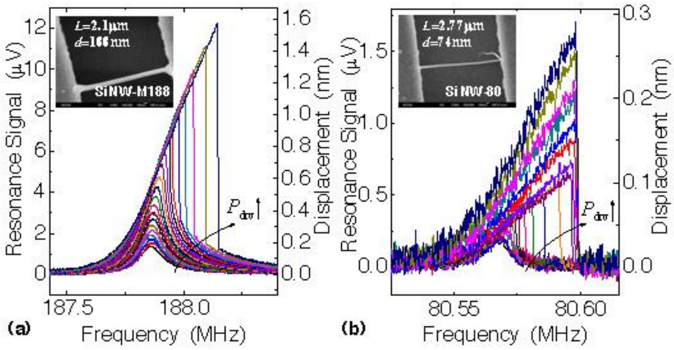

*Figure 2. Measured SiNW resonance signals with rms voltage and displacement, with data from **(a)** SiNW-M188 (B=6T, drive to SiNW P_{drv} from −61 to −41dBm, with +1dB step) and **(b)** SiNW-80 (B=8T, P_{drv} from −74 to −54dBm, with +1dB step), all at T=25K.*

As an example, for SiNW-M188 the measured onset of nonlinearity occurs at $P_{drv}\approx-47$dBm. At the 1dB compression point the resonance peak rms voltage is $V_{-1dB}\approx6\mu$V, yielding a midpoint rms displacement $a_{-1dB}\approx0.771$nm. This is in excellent agreement with the theoretical estimation 0.775nm given by $a_{-1dB}=\left(0.745\omega_0 L^2/\pi^2\right)\sqrt{\sqrt{3}\rho/2E_Y Q}$ using material and device parameters detailed in [4].

3. NOISE IN RESONANCE DETECTION

Essentially the transduction of a SiNW resonator is to electronically detect the displacement of the vibrating SiNW. For noise arising from various origins in the course of detection, we consider the *intrinsic* and *extrinsic* noise separately.

Thermomechanical motion of the device sets a fundamental limit for the intrinsic noise. The thermomechanical displacement power spectral density (per unit bandwidth) is given by

$$S_x = 4k_B TQ/\left(M_{eff}\omega_0^3\right). \qquad (1)$$

This corresponds to a force spectral density

$$S_F = 4k_B TM_{eff}\omega_0/Q. \qquad (2)$$

The displacement and force sensitivities of SiNW-M188 are $S_x^{1/2}\approx6.8$fm/Hz$^{1/2}$, $S_F^{1/2}\approx171.5$aN/Hz$^{1/2}$, respectively, limited by the thermomechanical noise. Combining this displacement noise floor and the onset of nonlinearity determines the device intrinsic dynamic range $DR_{intrs}\approx101$dB (for 1Hz bandwidth). In magnetomotive transduction, the intrinsic electronic noise floor is then set by the electromotive force (emf) induced by the SiNW's thermomechanical motion in the B field,

$$S_{Vemf} = 4k_B TQ\eta^2 B^2 L^2/\left(M_{eff}\omega_0\right). \qquad (3)$$

For SiNW-M188, we have $S_{Vemf}^{1/2}\approx0.053nV/Hz^{1/2}$. In practical measurements, however, intrinsic thermomechanical noise of this low level and the large intrinsic DR may not be readily accessible if other noise comes into play. The intrinsic noise specs of another metalized 200MHz resonator (SiNW-M200) and two pristine devices, SiNW-80 and SiNW-215, are collected in Table 1.

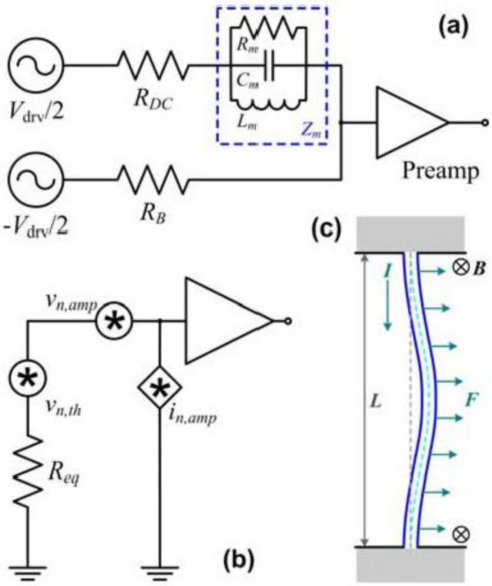

*Figure 3. **(a)** Simplified detection circuit with SiNW's electromechanical impedance. **(b)** Equivalent circuit for detection noise analysis. **(c)** Illustration of the magnetomotive transduction.*

Extrinsic noise arises when electromechanical transduction and electronic amplification are implemented. Figure 3 (a) depicts the simplified circuit for SiNW resonance detection. Here $Z(\omega)=R_{DC}+Z_m(\omega)$ is the total impedance of the SiNW resonator, with $Z_m(\omega)$ being the equivalent electromechanical impedance modeled by a

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parallel *LCR* tank. VHF SiNWs generally have $|Z_m| << R_{DC}$; hence, ideally the bridge resistor R_B should equal R_{DC} to achieve efficient background nulling. Figure 3 (b) shows a simplified model for analyzing electronic noise. Near resonance, we consider Johnson noise from $R_{eq}=(R_{DC}+R_m)||R_B$, and preamplifier noise which is mostly dominated by voltage noise and thus can be approximated by using the equivalent noise temperature T_N calibrated against R_S=50Ω. Therefore the total voltage noise spectral density referred to the input of preamplifier is

$$\begin{aligned} S_v &= 4k_B T R_{eq} + \left(v_{n,amp}^2 + i_{n,amp}^2 R_{eq}^2\right) \\ &\approx 4k_B T R_{eq} + 4k_B T_N R_S \end{aligned} \quad (4)$$

We estimate $S_v^{1/2} \approx 0.29\mathrm{nV/Hz}^{1/2}$ for SiNW-M188. Equivalently this translates back into an effective displacement noise floor of $S_{x,eff}^{1/2} \approx 36.9\mathrm{fm/Hz}^{1/2}$, which is 5.4 times of the thermomechanical displacement. This noise mismatch causes a 14.6dB loss in *DR* and hence the available *DR* is DR_{avl}=86.4dB. Such extrinsic noise results from the four SiNWs are summarized in Table 1.

Table 1. Noise characteristics of the VHF SiNW resonators and the performance of SiNW-NEMS-PLL.

Spec \ Device	SiNW-M188	SiNW-M200	SiNW-80	SiNW-215
f_0 (MHz)	187.9	199.7	80.57	215.4
a_C (nm)	1.47	2.03	0.48	0.80
$S_x^{1/2}$ (fm/Hz$^{1/2}$)	6.8	4.5	77.1	13.7
$S_{Vemf}^{1/2}$ (nV/Hz$^{1/2}$)	0.053 (*B*=6T)	0.040 (*B*=6T)	0.45 (*B*=8T)	0.13 (*B*=8T)
DR_{intrs} (dB)	101.1	107.6	70.4	89.8
$S_F^{1/2}$ (aN/Hz$^{1/2}$)	171.5	246.3	35.3	74.6
R_{DC} (Ω) (300K)	70.8	58.7	3.616k	3.135k
R_{DC} (Ω) (25K)	42.1	34.9	2.817k	2.343k
R_m (Ω)	3.9	2.2	283.8	23.7
R_{eq} (Ω)	19.8	19.7	1334.9	1286.4
T_N (K)	20	18.8	32	18
$S_v^{1/2}$ (nV/Hz$^{1/2}$)	0.29	0.28	1.39	1.35
$S_{x,eff}^{1/2}$ (fm/Hz$^{1/2}$)	36.9	31.7	236.7	141.1
DR_{avl} (dB)	86.4	90.6	60.6	69.5
$\Re$ (Hz/zg)	2.1	1.4	1.7	6.3
$\langle \delta f_0/f_0 \rangle_{\tau=1s}$ (ppm)	0.147	0.182	1.45	1.34
δM (zg)	13.3	25.5	67.9	45.9

4. SiNW RESONATOR PHASE NOISE

Phase noise of SiNW resonators could be characterized either in self-sustaining oscillation mode or with a phase-locked loop (PLL). We have built a SiNW-NEMS-PLL system (Figure 4) by embedding the SiNW resonance detection circuitry into a low-noise PLL where a voltage-controlled oscillator (VCO) (with superior noise performance and greater stability than the devices under test) is employed to drive the resonator and to provide the reference signal for phase-locking.

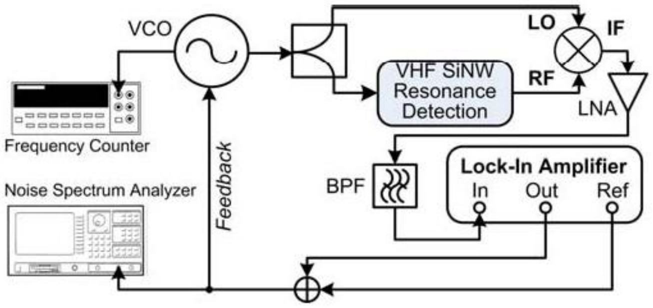

Figure 4. VHF SiNW-NEMS-PLL circuit diagram for characterizing the phase noise and frequency stability of SiNW resonators.

Figure 5 demonstrates measured phase noise from SiNW-M188-PLL, in comparison with the phase noise limits set by a few noise sources which can be approximately modeled. Ultimately, phase noise (in dBc/Hz) is limited by resonance phase diffusion due to device thermomechanical fluctuations,

$$S_\phi(f) = 10\log\left(k_B T / \left(P_C Q^2\right) \cdot \left(f_0 / f\right)^2\right), \quad (5)$$

where f is the offset frequency, and $P_C = E_C\omega_0/Q$, $E_C = M_{eff}\omega_0^2 a_0^2$. The best phase noise limit can be approached when the device is operating at max available rms displacement $a_0 = 0.745 a_C/\sqrt{2}$. Consider the resistor Johnson noise and amplifier noise in detection, both surpass the device thermomechanical noise and thus elevate the actual phase noise. We analyze the equivalent circuit for SiNW resonators by employing the similar techniques for developing resonant circuit phase noise models in [7]. Assuming that the preamplifier is also Johnson noise limited, we estimate the phase noise can be approximated by $10\log[(1+R_\phi/R_m)^2]$ in excess of that in (5), where R_ϕ is the effective resistance accounting for the influences of the resistive device and the nonideal preamplifier. The measured data suggests more excess phase noise as compared to the theoretical

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estimations. The roll-off at the mid-range offset frequencies is because of the limited bandwidth of the PLL (detection time constant is 10ms). The upward close-to-carrier noise is likely due to long-term drift. Characterization of the far-from-carrier noise requires further engineering of SiNW-PLL.

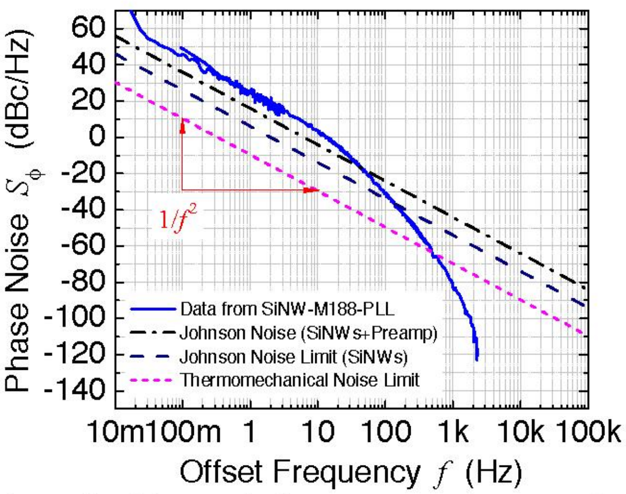

Figure 5. Measured phase noise performance from SiNW-M188-PLL and phase noise limits imposed by Johnson noise and thermomechanical noise.

5. FREQUENCY STABILITY

Time-domain measurements are made by using the system in Figure 4 with a high precision counter. Frequency stability is quantified by Allan deviation as a function of averaging time,

$$\sigma_A(\tau)=\sqrt{\frac{1}{2(N-1)}\sum_{i=1}^{N}\left(\frac{\bar{f}_{i+1}-\bar{f}_i}{f_0}\right)^2}\,. \qquad (6)$$

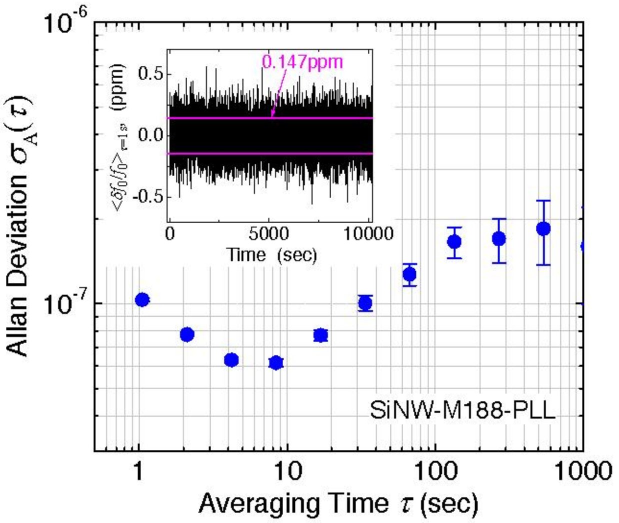

Figure 6. Measured Allan deviation as a function of averaging time for SiNW-M188. ***Inset:*** *Fractional frequency fluctuation with 1s averaging time.*

As shown in Figure 6, Allan deviation as low as 6×10^{-8} is achieved with 4~10s averaging time for SiNW-M188. At $\tau\approx$1s averaging time, the rms instantaneous fractional frequency fluctuation is 0.147ppm. When the system is exploited for resonant mass sensing at 1s averaging time, this frequency stability level, combined with the device's high mass responsivity [8] $\Re\approx$2.1Hz/zg, leads to a 13.3zg (1zg=10^{-21}g) mass sensitivity. This excellent performance is comparable to that of the state-of-the-art top-down VHF NEMS resonators [5]. Frequency stability and sensitivity from other SiNWs are also listed in Table 1. Phase noise and frequency instability are higher in pristine SiNWs than in metalized ones is because of more excessive Johnson noise injection.

6. CONCLUSIONS

We have investigated noise in the electronic detection of VHF SiNW NEMS resonators with magnetomotive transduction. The detection is currently limited by extrinsic electronic noise. Noise match to the intrinsic thermomechanical noise of VHF SiNW resonators is intriguing and challenging. We have characterized phase noise and frequency stability of SiNWs via integration with PLL systems. Excellent short-term stability and sensitivity are achieved. Moreover, our noise analyses have delineated routes for further optimization and engineering of SiNW-based low-noise resonators, oscillators, and sensors.

REFERENCES

[1] G.F. Zheng, C.M. Lieber, *et al.*, *Adv. Mater.*, vol. 16, pp. 1890-1893 2004.
[2] F. Patolsky, C.M. Lieber, *Materials Today,* vol. 8, pp. 20-28, 2005.
[3] R.R. He, P.D. Yang, *et al.*, *Adv. Mater.*, vol. 17, pp. 2098-2102, 2005.
[4] X.L. Feng, R.R. He, P.D. Yang, M.L. Roukes, *Nano Lett.*, vol. 7, 2007 (*to be published*).
[5] Y.T. Yang, C. Callegari, X.L. Feng, K.L. Ekinci, M.L. Roukes, *Nano Lett.*, vol. 6, pp. 583-586, 2006.
[6] C.T.C. Nguyen, *IEEE Trans UFFC*, vol. 54, pp. 251-270, 2007.
[7] A. Hajimiri, T.H. Lee, *IEEE JSSC*, vol. 33, pp. 179-194, 1998.
[8] K.L. Ekinci, Y.T. Yang, M.L. Roukes, *J. Appl. Phys.*, vol. 95, pp. 2682-2689, 2004.

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